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Code No. 1268/CORE

FACULTY OF SCIENCE

M. Sc. II – Semester Examination, December 2020

Subject : Mathematics / Applied Maths

Paper – I : Galois Theory

Time : 2 Hours **Max. Marks: 80**

Note: Answer any five questions. **(5x7=35 Marks)**

1 Let f(x) $\in$ F[x] be a polynomial of degree 2 or 3 then show that f(x) is reducible if and only if f(x) has a root in F.

2 Let f(x) $\in$ F[x] be a non-constant polynomial then there exists an extension E of F in which f(x) has a root.

3 Let f(x) $\in$ F[x] be a polynomial of degree $\geq$ 1 with α as a root then α is a multiple root if and only if f′(α) = 0.

4 Prove that the multiplicative group of non zero elements of a finite field is cyclic.

5 Prove that $G = G\left(\frac{Q(3\sqrt{2})}{Q}\right)$ is a group of Q-auto morphisms of Q$(3\sqrt{2})$ and |G| = 1.

6 Define fixed field of H. If $H = G\left(\frac{C}{R}\right)$ then prove that C_H=R.

7 Let F be a field and let U be a finite subgroup of the multiplicative group F* = F – {0} then prove that U is cyclic.

8 If a and b are constructible numbers then a $\pm$ b are also constructible.

PART – B

Note: Answer any three questions. **(3x15=45 Marks)**

9 (i) Let $f(x) = a_o + a_{1x} + \ldots + a_{n-1}x^{n-1} + x^n \in z[x]$ be a monic polynomial. If f(x) has a root $\alpha \in$Q then $a \in$ Z and a/a_0.

(ii) State and prove Eisenstein criterion.

10 Let E be an extension field of F and let u$\in$E be algebraic over F. Let p(x) $\in$ F[x] be a polynomial of the least degree such that p(u) = 0 then show that

(i) p(x) is irreducible over F

(ii) If g(x) $\in$ F[x] is such that g(u) = 0 the p(x)/ g(x)

(iii) there is exactly one monic polynomial p(x) $\in$ F[x] of least degree such that p(u) = 0

11 If f(x) $\in$ F[x] is irreducible over F then prove that all roots of f(x) have the same multiplicity.

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12 Let E be a finite extension of a field F then show that the following are equivalent
(i) E = F (α) for some $\alpha \in$ E
(ii) There are only a finite number of intermediate fields between F and E.

13 Let E be a finite separable extension of a field E then show that the following are equivalent
(i) E is a normal extension of F
(ii) F is a the fixed field of $G\left(\frac{E}{F}\right)$
(iii) [E : F] = | $G\left(\frac{E}{F}\right)$ |

14 Let F be a field of characteristic $\neq$ 2 or 3 Let f(x) = x^3 + bx + c be a separable polynomial over F. If f(x) is irreducible over F then the Galois group of f(x) is of order 3 or 6. Also then Galois group of f(x) is S_3 if and only if $\Delta = - 4b^3 - 27C^2$ is not a square in F, that is, there does not exist only element $\alpha \in$ F such that $\alpha^2 = \Delta$.

15 Let f(x) $\in$ F[x] is solvable by radical over F if and only if its splitting field E over F has solvable Galois group $G\left(\frac{E}{F}\right)$

16 (i) Prove that it is impossible to construct a square equal in area to the area of a circle of radius 1.
(ii) A regular n – g on is constructible if and only if ϕ (n) is a power of 2.

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Code No. 1270/CORE

FACULTY OF SCIENCE

M. Sc. II – Semester Examination, December 2020

Subject : Maths / Applied Maths

Paper – II : Lebesque Measure and Integration

Time : 2 Hours **Max. Marks: 80**

Note: Answer any five questions. **(5x7=35 Marks)**

1 Show that the smallest σ-algebra containing the class **C** of all closed sets in R is the class of all Borelsets $\mathfrak{B}$ in R.

2 Show that every continuous function f defined on a Lebesgue measurable set E is also Labesgue measurable.

3 Let ϕ: [0, 3] $\to$ R be defined by

$$\phi(x) = \begin{cases} \frac{1}{2}, & 0 \le x < 1 \\ \frac{-3}{2}, & 1 \le x < \frac{5}{2} \\ 5, & \frac{5}{2} \le x \le 3 \end{cases}$$ then evaluate $\int_{[0,3]} \phi$

4 Suppose f and g are non negative measurable functions defined on a measurable set E $\subseteq$ R. Such that f(x) $\le$ g(x) for all $x \in$ E. If g is Lebesgue integrable on E, then show that f is also Lebsegue integrable on E.

5 If f : [a , b] $\to$ R is a monotonic function then show that f is of bounded variation on [a, b].

6 Define vitali cover and give an example of a vitali cover for the set of all irrational numbers.

7 If f, g $\in L^1$ [0, 1] then show that f + g $\in L^1$ [0, 1] and $\|f+g\|_1 \le \|f\|_1 + \|g\|_1$

8 If f : [a, b] $\to$ R is absolutely continuous on [a, b], then show that f is continuous on [a, b].

PART – B

Note: Answer any three questions. **(3x15=45 Marks)**

9 (i) If $\{A_n\}_{n=1}^{\infty}$ is a sequence of subsets in R then show that $m^*\left(\bigcup_{n=1}^{\infty} A_n\right) \le \sum_{n=1}^{\infty} m^*(A_n)$.

(ii) Show that [0, 1] is an uncountable set.

10 Suppose E $\subseteq$ R. Then show that the following statements are equivalent.

(i) E is Lebesgue measurable

(ii) To each $\in$ > 0, there is an open set O such that $O \supseteq E$ and m*(O – E) < $\in$

(iii) There is a G_δ - set G such that G $\supseteq$ E such that m* (G – E) = 0.

11 Suppose f and g are bounded measurable functions defined on E where m (E) < ∞. Then show that $\int_E f + g = \int_E f + \int_E g$

12 State and prove Fatou's lemma.

13 Suppose $\{f_n\}$ is a sequence of measurable functions defined on E and converge in measure to f. Then show that there exists a subsequence $\{ f_{n_r} \}$ of the sequence $\{f_n\}$ such that $f_{n_r} \to f$ almost everywhere on E.

14 Suppose f is a monotonic increasing function defined on [a, b], then show that f is differentiable almost everywhere on [a, b] and also f' is Lebsegue integrable on [a, b] and $\int_{[a,b]} f' \leq f(b) - f(a)$

15 Suppose f is a bounded measurable function on [a , b] and $F(x) = \int_{[a,x]} f + F(a)$ then show that F is differentiable on [a, b] and $F'(x) = f(x)$ for all $x \in$ [a, b].

16 Let $f \in L^p$ [0, 1] and $g \in L^q$ [0, 1] where p, q are such that $\frac{1}{p} + \frac{1}{q} = 1$ and $1 < p < \infty$. Then show that $fg \in {}^1[0, 1]$ and $\int_{[0,1]} | fg | \leq \| f \|_p . \| g \|_q$.

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Code No. 1272/CORE

FACULTY OF SCIENCE

M. Sc. II – Semester Examination, December 2020

Subject : Mathematics / Applied Mathematics / Maths with Computer Science

Paper – III : Complex Analysis

Time : 2 Hours **Max. Marks: 80**

Note: Answer any five questions. **(5x7=35 Marks)**

1 Prove that $f(z) = e^x e^{-iy}$ is no where differentiable.

2 Find all roots of the equation $\log(z) = \frac{i\pi}{2}$.

3 Suppose C is the boundary of the triangle with vertices at the points 0, 3i and -4 oriented counter clockwise. Then prove that $\left| \int_C (e^z - \bar{z})dz \right| \le 60$.

4 Compute $\int_{|z|=1} \tan z \, dz$.

5 Let $\{z_n\}$ be a sequence converging to z. Prove that there exists a number M > 0 such that $|z_n| \le M \ \forall$ n.

6 Let C be the circle |z| = 3 in the positive sense. Compute $\int_C \frac{e^{-z}}{z^2} dz$.

7 State and prove Jordan's lemma.

8 Let $T(z) = \frac{az+b}{cz+d} (ad - bc \ne 0)$ be a non-identity bilinear transformation. Prove that $T^{-1} = T$ if and only if d = - a.

PART – B

Note: Answer any three questions. **(3x15=45 Marks)**

9 (i) State and prove the reflection principle.
 (ii) Find $|\cos z|^2$ in terms of x and y.

10 (i) Derive necessary conditions for a function to be differentiable at a point.
 (ii) Find a harmonic conjugate of u = $2x - x^3 + 3xy^2$.

11 (i) Suppose C is the boundary of the square with vertices at the points 0, 1, 1 + i , i positively oriented. Compute $\int_C \pi e^{\pi \bar{z}} dz$.

 (ii) Let C be the arc $z = 2e^{i\theta}, \theta \in [0, \frac{\pi}{2}]$. Prove that $\left| \int_C \frac{z+4}{z^2-1} dz \right| \le \frac{6\pi}{7}$.

12 State and prove Cauchy-Goursat theorem.

13 (i) State and prove Laurent's theorem.
 (ii) Prove the uniqueness of Laurent's series.

14 (i) State and prove Cauchy residue theorem.

(ii) Compute $\int_{|z|=1} \frac{1-\cos z}{z^2} dz$.

15 (i) State and prove Rouche's theorem.
(ii) Prove the fundamental theorem of algebra using Rouche's theorem.

16 Evaluate $\int_0^{\pi} \frac{\cos 2\theta}{1-2a\cos\theta + a^2} d\theta \quad (-1 < a < 1)$.

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